Review of Numerical Methods for the Analysis of Arbitrarily-Shaped Microwave and Optical Dielectric Waveguides

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Abstract —This paper presents a review of the numerical methods for the analysis of the homogeneous and inhomogeneous, isotropic and anisotropic, microwave and optical dielectric waveguides with arbitrarily-shaped cross sections. The characteristics of various methods are compared, and a set of qualitative criteria to guide the selection of an appropriate method for a given problem is proposed. The main approaches discussed are those of point matching, integral equations, finite difference, and finite element.

I. Introduction

The rapid growth in recent years in the millimeter-wave, optical fiber, and integrated optics arts has included the introduction of arbitrarily-shaped dielectric waveguides, which in many cases also happened to be arbitrarily inhomogeneous and/or arbitrarily anisotropic. This variety occurs either as a design preference or due to actual manufacturing processes of dielectric waveguides operating in the microwave-to-optical frequency spectrum. Most of such cases of waveguide arbitrariness do not lend themselves to analytical solutions. Many scientists have, therefore, given their attention to the project of constructing numerical methods that solve the arbitrarily-shaped dielectric waveguide, which may be anisotropic and/or transversely inhomogeneous. In this paper, an attempt is made to provide a brief outline and selected bibliography of such contributions presented in the literature.

In varying degrees, modern developments in the dielectric and optical waveguide arts have benefited from the already matured metallic waveguide art. It is specifically noticeable that most of the numerical methods to solve the former problems were developed by adapting the corresponding approaches that proved successful in solving the latter. It seems appropriate, therefore, to refer to two review papers, by Davies [1] and Ng [2], of the metallic waveguide numerical methods, which may help establish certain useful analogies with this review.

After defining the problem and characterizing the methods of solution in Section II, the paper devotes Sections III–V to the numerical methods that solve the isotropic homogeneous, the isotropic inhomogeneous, and the anisotropic guides, respectively. Section VI proposes a set of qualitative criteria to judge such methods. As the paper title indicates, only the methods which are capable of solving the arbitrarily-shaped guide will be considered here, although a number of numerical methods have been developed for specific cross-sectional shapes, for example, the circular fiber with radially varying refractive index [3].

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II. Characterization of the Problem and the Methods of Solution

We are considering here the methods for solving numerically the multilayer longitudinally-uniform dielectric waveguide of arbitrarily-shaped cross section. As our most general case, the guide is anisotropic and transversely inhomogeneous. The method should find a numerical solution to Maxwell's equations

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \tag{1}$$

valid over each layer (region) R_i, and subject to the boundary conditions, namely, the tangential field components must be continuous across the boundary C_i between any two layers i and $i+1$. A time variation of $\exp(-j\omega t)$ is adopted and omitted throughout the paper.

The numerical methods of solving (1) differ in the following respects. Firstly, some methods are aimed at direct numerical solution of (1), their equivalent integral forms, or any of their special-case reduced forms; but the majority of methods aim at transforming such differential or integral equations, through various mathematical modeling schemes, into a system of linear equations solvable by standard matrix techniques. Secondly, the method may approximate the field over each dielectric layer of the waveguide, either at discrete points, employing finite differences, or by an expansion valid over the entire layer, such as in the point-matching method, or by a set of expansions, where each is valid over a subregion of each dielectric layer, such as in the finite-element method. Thirdly, the methods differ in the way they deal with the infinite extent of the waveguide cross section. For example, the point-matching and some finite-element approaches take advantage of the fact that the field decays in the radial

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direction away from the guide, while some other finite-element approaches impose various kinds of boundary conditions at some optimized distance far enough from the guide.

In order to improve the accuracy or efficiency of a numerical technique, it proved advantageous in some cases to combine two or more numerical methods in the solution. For example, a point-matching method may be supplemented by a moment method to improve the accuracy of matching the fields along the boundary C. Also, a variational formulation may be useful to attach to several approximation techniques as a part of the error minimization procedure. It is interesting to note that many authors have done just that, and several hybrid approaches have been elegantly constructed. For us to classify such approaches into groups, with as little confusion as possible, we have chosen to identify each approach according to its constituent method which is either dominant in the solution procedure or was responsible for the breakthrough in the literature.

III. The Isotropic Homogeneous Waveguide

In the special case of an isotropic homogeneous waveguide, (1) can be reduced into the scalar wave equation

$$\nabla^2\phi + k_i^2\phi = 0 \tag{2}$$

where ∇^2 is the transverse Laplacian operator, ϕ is the scalar field E_z or H_z, and k_i is the wavenumber of the ith layer defined by

$$k_i^2 = \begin{cases} k_0^2 n_i^2 - \beta^2, & k_0 n_i \geqslant \beta \\ \beta^2 - k_0^2 n_i^2, & k_0 n_i \leqslant \beta \end{cases} \tag{3}$$

where $k_0 = \omega(\mu_0\epsilon_0)^{1/2}$ is the free-space wavenumber, β is the propagation constant, and n_i is the refractive index of layer i.

A. *The Point-Matching Method*

The point-matching method is one of the oldest and simplest techniques for the solution of the isotropic homogeneous dielectric waveguide with arbitrary cross section. Its application to the two-layer rectangular cross-section guide was shown first by Goell [4]. The method maintains that a good approximation to the scalar field in (2) is

$$E_{z1} = \sum_{n=0}^{\infty} A_n \sin(n\theta + u_e) J_n(k_1 r) \exp(j\beta z) \tag{4a}$$

$$H_{z1} = \sum_{n=0}^{\infty} B_n \sin(n\theta + u_h) J_n(k_1 r) \exp(j\beta z) \tag{4b}$$

in the interior region R_1, and

$$E_{z2} = \sum_{n=0}^{\infty} C_n \sin(n\theta + u_e) K_n(k_2 r) \exp(j\beta z) \tag{4c}$$

$$H_{z2} = \sum_{n=0}^{\infty} D_n \sin(n\theta + u_h) K_n(k_2 r) \exp(j\beta z) \tag{4d}$$

in the infinite exterior region R_2. By matching the tangential fields at optimally selected N points around the boundary, truncating the expansions in (4) at $n = N$, we obtain a system of the linear equations in the unknown coefficients A_n, B_n, C_n, and D_n. By applying the condition of nontrivial solution, a characteristic equation in β is obtained and solved for possible eigenvalues. The original matrix equation is then solved for each mode eigenfunction by standard matrix techniques.

Improved results were reported by Cullen *et al.* [5] who followed Goell's approach, but rotated the grid of equi-angularly-spaced matching points in order to place a matching point at the corner of a rectangular dielectric waveguide. Later, Cullen and Ozkan [6] showed that the fundamental mode fields, obtained by point matching, yield satisfactory results even when used to compute the coupling coefficients between two rectangular rods. Again, Goell's approach was employed by Yamashita *et al.* [7] to obtain a modal analysis of the elliptical, egg-shaped, and chipped-circle fibers, and by Saad [8] to resolve a dispute among various analytical solutions concerning the higher order mode cutoff of the elliptical fiber with large eccentricity.

The above point-matching approach was extended by Yamashita *et al.* [9] to solve the composite (three-region) dielectric waveguide, and was later modified to treat some cases of composite nonconvex shapes [10]. In the above attempts [4]–[9], the accuracy of the point-matching solution was proven either experimentally or by comparison with other confirmed solutions.

When the point-matching approach was originally proposed by Goell, there were many unanswered questions about its physical justification, the legitimacy of its straightforward unconditional application, the validity of expansions such as (4) in representing the interior and exterior solutions everywhere on C, the convergence of its eigenvalue and eigenfunction solutions to the correct values as N increases to infinity, and the sufficiency of its accuracy to engineering applications.

In addressing these questions, James and Gallett [11] had examined certain criteria as a basis for the validity and accuracy of the point-matching method. Expansion (4), they conclude, is not valid unless the boundary C is radially single-valued and free from prominent angular periodicities. An illuminating follow-up discussion on their claims is given by Bates *et al.* [12]. Further work by Hafner and Ballisti [13] proves that the validity of (4) depends on the structure of region R_1 rather than the shape of the boundary C.

B. *Variational and Integral Approaches*

Here, three phenomena may be utilized. First, a variational formulation that is "stationary" about the correct solution may be adopted to minimize the error in field modeling. Second, by deriving an appropriate integral representation of the field, satisfying the boundary conditions, is guaranteed. Third, by using a simple trial field or an expansion, the integral equations are reduced to a set of linear equations to be solved by standard computer methods.

James and Gallet [14] have proven that although some configurations, like the triangular fiber, may not lend themselves to an accurate point-matching solution, such solution can still be useful if included as a trial field in a variational formulation that utilizes an integral representation of a correction field.

By implementing the variational technique for the E-field integral equation, Keuster and Pate [15] obtained a solution for the fundamental mode. Their dispersion curves, obtained by assuming a constant trial field within the core, seem to be accurate only at lower frequencies.

The most powerful and versatile method in this category may be that of Eyges *et al.* [16]. By providing an expansion of the field in terms of appropriate basis functions, such as (4), in the integral representations, a matrix equation is formed with elements involving these basis functions in line integrals that are taken over the boundary C. Thus, the need to explicitly match the interior and exterior solutions across C is eliminated. Their paper documents accurate solutions for several higher order modes in rectangular and elliptical guides.

C. *Combined Integral and Differential Equation Approaches*

An interesting combination of integral and differential equation approaches was developed by Williams and Cambrell [17], [18], where an integral equation is derived to describe the impedance boundary conditions on an auxiliary boundary located just outside region R_1. Meanwhile, Maxwell's equations were transformed into an eigenvalue matrix equation using the finite-difference method in [17], and using the moment method in [18].

IV. The Isotropic Inhomogeneous Waveguide

In the case of an isotropic waveguide with transverse inhomogeneity, the solution is required to satisfy a reduced form of (1), namely, the vector wave equation

$$\nabla^2 \boldsymbol{A} + k_0^2 \epsilon_r(x, y)\boldsymbol{A} - \frac{\nabla \epsilon}{\epsilon} \nabla \cdot \boldsymbol{A} = 0 \qquad (5)$$

which may be, for convenience, rearranged into a set of two coupled second-order differential equations in the vector potential transverse components A_x and A_y.

A. *The Finite-Difference Method*

The finite-difference method is the oldest and perhaps the most commonly used technique for the solution of boundary value problems [1]. Except for the one-dimensional homogeneous case [17], however, the method has not been applied to dielectric waveguides until recently. Because it divides the waveguide cross section into a large number of subregions, the method lends itself "naturally" to the solution of inhomogeneous guides.

Decotignie *et al.* [19] developed a finite-difference method for the solution of (5). They selected a nonregular grid so as to place the external boundary conditions (zero field components, in their choice) as far as desired, without increasing unnecessarily the number of mesh nodes. The finite-difference method was more advantageously defined, via a variational approach, by Schweig and Bridges [20]. Here, the guide is enclosed in a conducting box which is sufficiently large so that it does not perturb the modes. Instead of trying to solve (5), the simpler wave equation (2) is considered because it is approximately valid over each subregion. In both applications [19], [20], the authors have proven one important advantage of the finite-difference method over the finite-element method, namely, the former is free from the troublesome problem of "spurious numerical modes" which are generated by the numerical technique while they actually do not represent physical modes of the waveguide.

B. *The Finite-Element Method*

Despite its short history, the finite-element method has grown to one that offers probably the most powerful and efficient numerical solution of the most general (i.e., arbitrarily-shaped, inhomogeneous, and anisotropic) optical waveguide problem. Here, the waveguide cross section is divided into a large number of triangles (elements), and the field in each element is represented by a polynomial, then the field continuity conditions are imposed on all interfaces between the different elements. By employing a variational expression for the Maxwell's equations, or their reduced forms (2) or (5), an eigenvalue matrix equation is obtained and solved using standard methods.

While the use of a variational formulation in the finite-difference method is optional, though preferred, it is a necessary step in the finite-element method. It is interesting to note that it is the variational approach that brings close together the finite-difference and finite-element techniques [1]. As pointed out in [20], the two techniques become equivalent for one-dimensional problems and for two-dimensional problems with rectangular boundaries.

Taking advantage of the fact that the field of the mode above cutoff decays in the exterior region, Yeh *et al.* [21] introduced a finite-element method which incorporates into the field approximation in the boundary elements an exponential decay factor, to be determined heuristically. Such a factor is employed to approximate the infinite exterior region by an equivalent closed region with imposed boundary conditions. Their solution, though limited to the fundamental mode, was demonstrated for numerous shapes of homogeneous waveguides as well as the diffused channel waveguide.

At mode cutoff, the exterior field is either constant or decaying very slowly. While this phenomenon makes the decay factor modeling [21] impossible or inaccurate at cutoff, it was alternatively employed by Chiang [22] to implement a Neumann boundary condition at infinity, and to seek a new finite-element formulation based on direct solution of the wave equation at cutoff. By limiting the solution to small variations in $n(x, y)$, the scalar wave equation (2) becomes applicable. Also, by assuming homogeneous cladding, (2) then, as applied to the exterior region, reduces to Laplace equation at cutoff. This allows a much simpler finite-element algorithm [23] to be used, which in turn significantly improves the efficiency of modeling the infinite exterior region by a finite region.

C. *Semi-Numerical Approaches*

The finite-differences and finite-element methods represent a totally numerical solution that approximately satisfies Maxwell's equations over each of the regions R_i. In contrast, differential and integral equation approaches attempt to find a semi-numerical solution that satisfies Maxwell's equations exactly over R_i but approximately over the boundary C. In this class of approaches, Maxwell's equations are transformed into an eigenvalue equation through some formal analytical procedure.

By utilizing the integral equations of the field scattered by a dielectric cylinder, and employing the procedure of the extended boundary condition, Morita [24] solved the problem of inhomogeneous dielectric waveguide surrounded by free space. The advantage here, as in other integral formulation approaches, is that the two-dimensional problem is reduced to a one-dimensional problem, and numerical integration is performed only over the boundary C.

Schelkunoff [25] has shown that a valid method of solving Maxwell's equations for the waveguide problem is by transforming them first into a set of generalized telegraphist's equations for the mode voltages and mode currents. Some 25 years later, Ogusu [26] applied such a method to the inhomogeneous dielectric waveguide by enclosing it in a hypothetical rectangular electric boundary. Special application of the method to the Y-shaped homogeneous guide was later reported by Shinonaga and Kurazono [27].

V. The Anisotropic Waveguide

In the case of an anisotropic waveguide, one may derive from (1) the following vector wave equation:

$$\nabla \times \left([\epsilon_r]^{-1} \nabla \times \boldsymbol{H} \right) - k_0^2 \boldsymbol{H} = 0 \qquad (6)$$

where $[\epsilon_r]$ is the relative permittivity tensor. In this section, we will consider both the homogeneous and inhomogeneous waveguides, and review available solutions for (1) and (6).

A. *The Integral Equation Method*

Upon using the free-space Green's function, de Ruiter [28] transformed (1) into a system of homogeneous integral equations. The latter is then solved using a combination of the methods of moments and point matching. The theory is developed for transversely inhomogeneous waveguide with complex and frequency-dependent permittivity and permeability. The practical case of a lossy, uniaxially anisotropic medium, with a longitudinal extraordinary axis, is included.

B. *The Finite-Element Method*

Before surveying the finite-element methods for the anisotropic waveguide, a word about spurious solutions is due. As pointed out by several authors, spurious solutions do not exist in the scalar finite-element formulation because the operator there is positive definite. In contrast, the anisotropic problem requires a vector finite-element formulation where the operator is no longer positive definite. This and few other possible causes for the presence of such spurious solutions were recently investigated by Rahman and Davies [29].

Mabaya *et al.* [30] developed a finite-element method for the case of an anisotropic guide of arbitrary cross section and index variation, and an anisotropic substrate region. By considering a diagonal permittivity tensor, a variational expression for (1) in terms of E_z and H_z was possible. In order to model the infinite transverse extent of the waveguide, the method imposes an artificial Dirichlet boundary condition at an optimally-determined distance from the guide. The authors apparently succeeded in reducing the number of spurious numerical modes from the eigenvalue matrix solution (though admittedly the procedure has no mathematical foundation) by explicitly enforcing the continuity of the tangential components of the transversal fields at the boundaries by means of Lagrange multipliers. Recently, Koshiba *et al.* [31] extended this approach to the case of a permittivity tensor with nonzero, but relatively very small, off-diagonal elements.

In order to deal with arbitrary permittivity tensors, Rahman and Davies [32] presented a vector $\boldsymbol{H}$-field formulation of the problem, i.e., a variational expression for (6) in terms of all three components of $\boldsymbol{H}$. Because transverse fields are most important than axial fields in optical dielectric waveguides, even the special cases of [30] and [31] could be more accurately computed by [32]. To identify spurious solutions, [32] utilizes an earlier computer experiment finding [33] that such solutions do not satisfy the relation $\nabla \cdot \boldsymbol{H} = 0$.

A higher objective is, of course, to eliminate, not just identify, spurious solutions. Rahman and Davies [29] did just that by introducing a "penalty function" into the vectorial finite-element formulation. (This resulted in a useful by-product, namely, an improved quality of the physical field solutions.) Another approach was presented by Koshiba *et al.* [34], [35], who reformulated the functional for (6) such that $\nabla \cdot \boldsymbol{H} = 0$ is guaranteed in the whole interior region.

VI. Criteria for Method Selection

In judging the appropriateness of various numerical methods to solve a particular cross-sectional shape, possibly with given inhomogeneity and/or anisotropy, in a prescribed frequency range, one has to relate the specific problem to the following assessment criteria.

1) The ability of the method to deal with more than two homogeneous dielectric layers. Simple point matching, for example, has been implemented for only two-layer waveguides, and extended to composite (three-region) guides. Although it seems possible to extend the method to treat a larger number of regions (and perhaps inhomogeneous

guides too), the accuracy and efficiency of such an approach may be uncertain.

2) The accuracy of the method in modeling the dielectric boundaries and regions. Finite element and finite difference will yield more accurate results if applied to linear, rather than curved, boundaries. They will also yield much better accuracy, compared to point matching, for example, when applied to nonconvex shapes.

3) The accuracy of the method in specific frequency ranges. Most methods have one kind or another of built-in source of error that accelerates near cutoff. The methods developed specifically for the cutoff frequency, e.g., [22], do yield, as would be expected, the best results at such a frequency.

4) The sufficiency of accuracy of the results. For example, a method may be perfectly adequate to compute the characteristics of a single mode, but not adequate to compute the small differences between nearly degenerate modes.

5) The built-in restrictions in the method. For example, [22] is meant to solve the inhomogeneous fiber, but two key assumptions in the solution, namely, small variations in $n(x, y)$ and a homogeneous cladding, will limit the method's applicability.

6) Whether the method has a mechanism for generating spurious numerical solutions, and if so, whether the method can lend itself to some modification that could identify and/or eliminate such spurious solutions.

7) The degree of understanding and involvement required from the user of the method. While some methods can be realized as computer programs that solve a wide range of shapes, inhomogeneity, and/or anisotropy, others have to be implemented, with varying grades of difficulty, by the user.

8) The computational efficiency of the method, including its computer storage requirements.

VII. Conclusion

As the state of the dielectric waveguide numerical analysis art reflects progress in the last decade, it still covers thinly a wide range of new practical waveguide possibilities. The user of such methods does face a decreasing number of options as the design objectives or manufacturing processes introduce more general cases of inhomogeneity and anisotropy. Substantial new work is thus required to improve existing options of numerical methods, and create new competitive, if not better, ones.

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